# Wireless Communications

## EENG 5820

Lecture 10

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# 9.7 Capacity of Cellular Systems

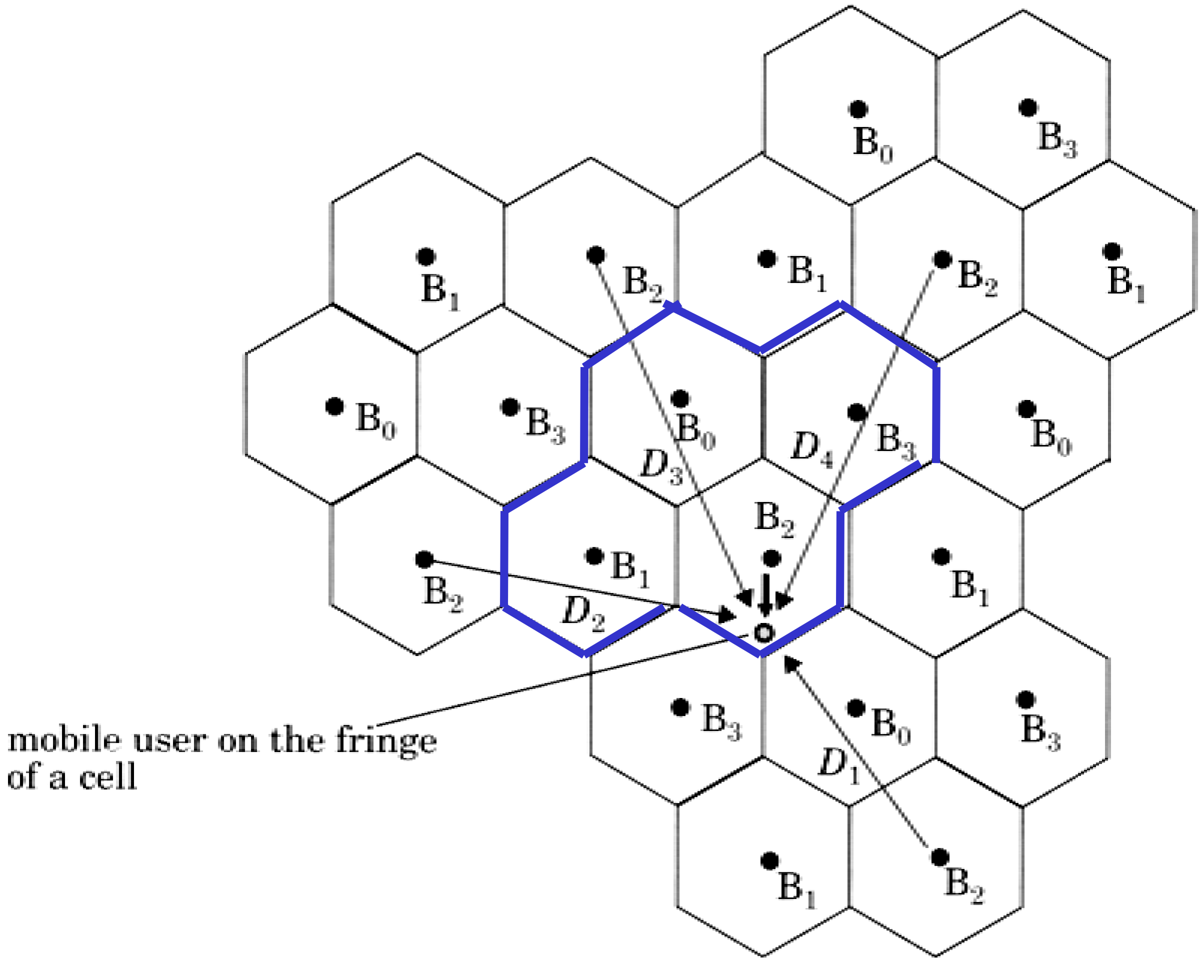


**Figure 9.11** Illustration of forward channel interference for a cluster size of $N = 4$. Shown here are four co-channel base stations which interfere with the serving base station. The distance from the serving base station to the user is $D_0$, and interferers are a distance $D_k$ from the user.



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# 9.7 Capacity of Cellular Systems

- **Channel capacity: the maximum number of channels or users than can be provided in a fixed frequency band**
  - Required carrier-to-interference ratio (C/I)
  - Channel bandwidth $B_c$.
- **Carrier-to-interference (C/I)**

$$\frac{C}{I} = \frac{D_0^{-n_0}}{\sum_{k=1}^{M} D_k^{-n_k}} \xrightarrow{\text{Approximation}} \frac{C}{I} = \frac{D_0^{-n_0}}{6D^{-n}}$$

$n_k$ : path loss exponent

# 9.7 Capacity of Cellular Systems

- **When $D_0=R$, maximum interference, and minimum C/I is $(C/I)_{min}$, then**

$$\frac{1}{6}\left(\frac{R}{D}\right)^{-n} \geq \left(\frac{C}{I}\right)_{min} \xrightarrow{Q = D/R} Q = \left(6\left(\frac{C}{I}\right)_{min}\right)^{-1/n}$$

**Radio capacity:**

$$m = \frac{B_t}{B_c N} \text{ radio channels/cell} \xrightarrow{Q=\sqrt{3N}} m = \frac{B_t}{B_c \frac{Q^2}{3}} = \frac{B_t}{B_c\left(\frac{6}{3^{n/2}}\left(\frac{C}{I}\right)_{\min}\right)^{2/n}}$$

**$B_t$: total spectrum**
**$B_c$: channel bandwidth**


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# 9.7 Capacity of Cellular Systems

## Note:

- (C/I)min is lower in digital systems than analog system (12 dB for digital system and 18dB for FM)
- To compare different systems, an equivalent $(C/I)_{eq}$ I used. Lower $(C/I)_{min}$ means more capacity

For n = 4

$$m = \frac{B_t}{B_c \sqrt{\frac{2}{3}\left(\frac{C}{I}\right)_{\min}}}$$

$$\left(\frac{C}{I}\right)_{eq} = \left(\frac{C}{I}\right)_{\min} \left(\frac{B_c}{B'_c}\right)^2$$


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# 9.7 Capacity of Cellular Systems

## Example: Compare four systems for n=4

- System A: $B_c$ = 30kHz, $(C/I)_{min}$ = 18dB
- System B: $B_c$ = 25kHz, $(C/I)_{min}$ = 14dB
- System C: $B_c$ = 12.5kHz, $(C/I)_{min}$ = 2dB
- System D: $B_c$ = 6.25kHz, $(C/I)_{min}$ = 9dB

Consider: $B_c^{'} = 6.25kHz$

**System A:** $(C/I)_{eq} = 18 - 20\log(6.25/30) = 31.68dB$

**System B:** $(C/I)_{eq} = 14 - 20\log(6.25/25) = 26dB$

**System C:** $(C/I)_{eq} = 2 - 20\log(6.25/12.5) = -4dB$

**System D:** $(C/I)_{eq} = 9 - 20\log(6.25/6.25) = 9dB$

# 9.7 Capacity of Cellular Systems

- **Digital cellular systems**

$$\frac{C}{I} = \frac{E_b R_b}{I} = \frac{E_c R_c}{I}$$

$R_b$: bit rate
$R_c$: symbol rate

For n = 4

$$\frac{(C/I)}{(C/I)_{eq}} = \frac{(E_c R_c / I)}{(E_c^{'} R_c^{'} / I')} = \left(\frac{B_c^{'}}{B_c}\right)^2$$

↓ $R_c$ and $B_c$ is linear

$$\frac{E_c}{E_c^{'}} = \left(\frac{B_c^{'}}{B_c}\right)^3$$

# 9.7 Capacity of Cellular Systems

## For FDMA systems

$$m = \frac{B_t}{\frac{B_t}{M}\sqrt{\frac{2}{3}\left(\frac{C}{I}\right)_{\min}}}$$

$$C = E_b R_b$$

$$I' = I_0 B_c$$

$I_0$ : Interference per Hertz

## For TDMA systems

$$C' = E_b R_b^{'}$$

$$I' = I_0 B_c^{'}$$

$I_0$ : Interference per Hertz

# 9.7 Capacity of Cellular Systems

## Compare systems.

- FDMA: 3 channels, each is 10kHz, data rate 10kbps
- TDMA: 3 time slots, channel bandwidth 30kHz, data rate 30kpbs

$$C' = E_b R_b' = 3R_b E_b = 3C$$

$$I' = I_0 B_c' = 3I$$

**C/I is the same**

# 9.7.1 Capacity of Cellular CDMA

- **FDMA/TDMA is bandwidth limited**
- **CDMA Is interference limited**
- **Techniques to reduce interference**
  - Multisectorized antenna
  - Discontinuous Transmission Mode (DTX)
    - Voice signals duty: 3/8 in landline networks, ½ for mobile systems

# 9.7.1 Capacity of Cellular CDMA

- For a single-cell system with power control, all the signals on the reverse channel are received at the same power level at the base station.
- Let the number of users be N, then signal-to-noise ratio is

$$SNR = \frac{S}{(N-1)S} = \frac{1}{N-1}$$

Bit energy-to-noise ratio

$$\frac{E_b}{N_0} = \frac{S/R}{(N-1)(S/W)} = \frac{W/R}{N-1}$$

# 9.7.1 Capacity of Cellular CDMA

- **Consider the background thermal noise η**

$$\frac{E_b}{N_0} = \frac{W/R}{N-1} \rightarrow \frac{E_b}{N_0} = \frac{W/R}{(N-1)+(\eta/S)}$$

$$N = 1 + \frac{W/R}{E_b/N_0} - (\eta/S)$$

- **Consider voice activity α or sectorizing**

$$\frac{E_b}{N_0^{'}} = \frac{W/R}{(N_s-1)\alpha + (\eta/S)}$$

Ignoring noise

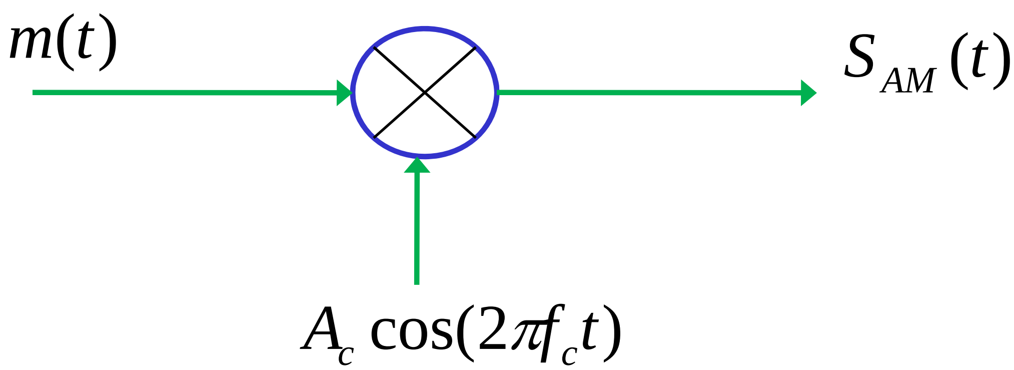
$$N_s = 1 + \frac{1}{\alpha}\left[\frac{W/R}{E_b/N_0^{'}}\right]$$

# Ch. 6 Modulation Techniques

- **Analog Communication**
  - AM: DSB, SSB
  - FM: constant envelope
- **Digital Communication**
  - Linear Modulation: PSK, PAM,
  - Constant Envelope Modulation: FSK

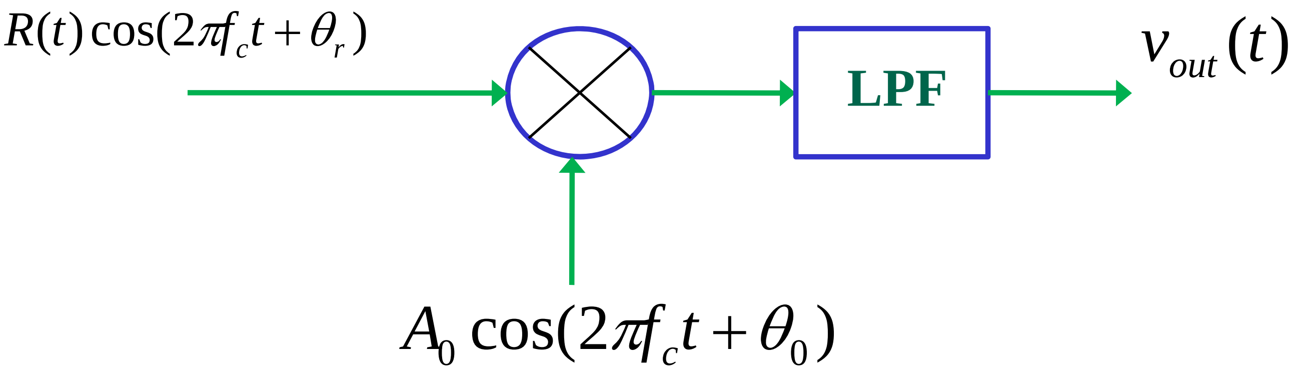
# 6.2 Amplitude Modulation

## Modulator




$$S_{AM}(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$$

## Demodulator



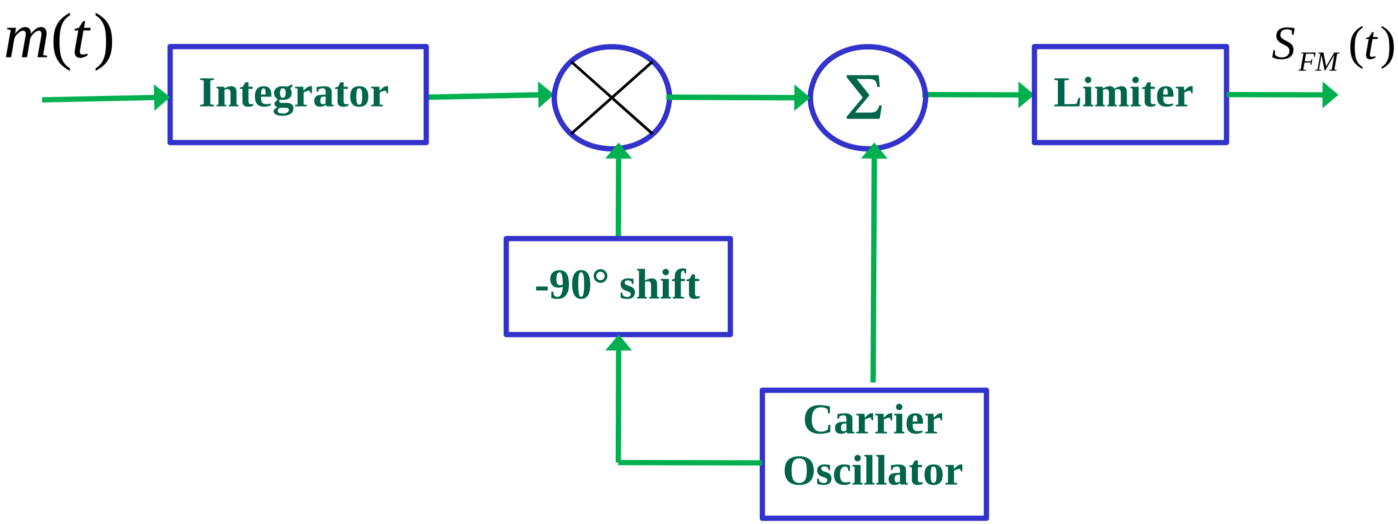

$$v_{out}(t) = \frac{1}{2} A_0 R(t) \cos(\theta_r - \theta_0)$$

## Modulator

$$S_{FM}(t) = A_c \cos(2\pi f_c t + \theta(t)) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]$$

$$S_{FM}(t) \cong A_c \cos 2\pi f_c t - A_c \theta(t) \sin 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t - A_c \left[2\pi \int_{-\infty}^{t} m(\tau)dt\right] \sin 2\pi f_c t$$



## Demodulator: Slope Detector

$$v_1(t) = V_1 \cos[2\pi f_c t + \theta(t)] = V_1 \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]$$

$$\Downarrow$$

$$v_2(t) = V_1\left[2\pi f_c + \frac{d\theta(t)}{dt}\right]\sin[2\pi f_c t + \theta(t)]$$

$$v_{out}(t) = V_1\left[2\pi f_c + \frac{d\theta(t)}{dt}\right] = V_1 2\pi f_c + V_1 2\pi k_f m(t)$$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## Demodulator: Quadrature Detector

**Phase-shift network**

$$\phi(f) = -\frac{\pi}{2} + 2\pi K(f - f_c)$$

**Output of phase-shift network with FM input**

$$v_\phi(t) = \rho A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau + \phi(f_i(t))\right]$$

$$f_i(t) = f_c + k_f m(t)$$



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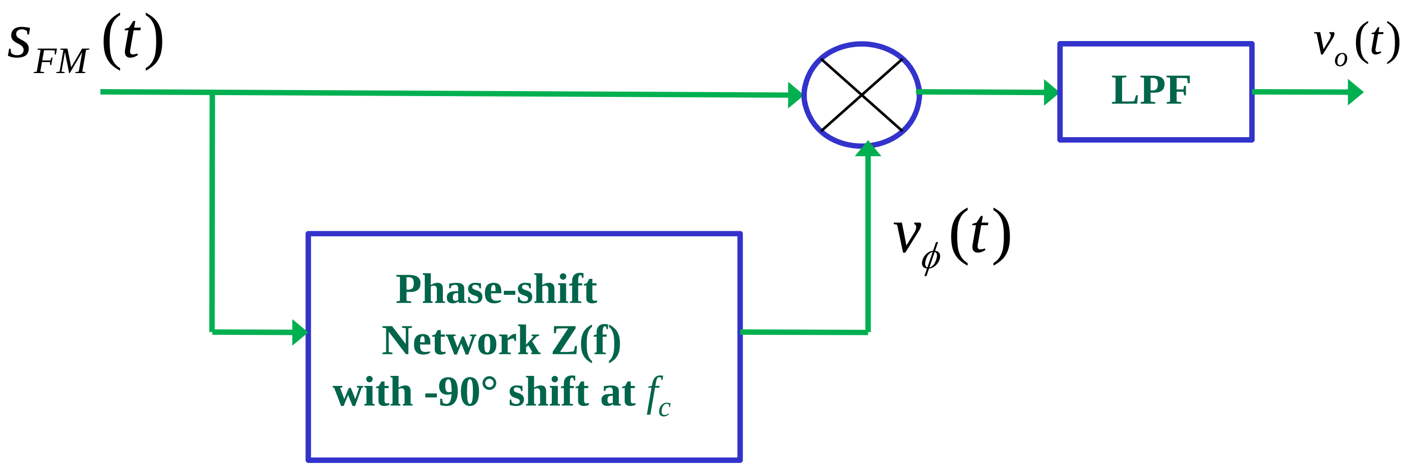
## Demodulator: Quadrature Detector




$$v_o(t) = \rho^2 A_c^2 \cos(\phi(f_i(t)))$$

$$= \rho^2 A_c^2 \cos(-\frac{\pi}{2} + 2\pi K(f_i(t) - f_c))$$

$$= \rho^2 A_c^2 \sin(2\pi K k_f m(t))$$

$$\rho^2 A_c^2 2\pi K k_f m(t) = Cm(t)$$

# 6.8 Binary Phase Shift Keying

$$S_{BPSK}(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_c) \Rightarrow \text{bit } 1$$

$$S_{BPSK}(t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_c) \Rightarrow \text{bit } 0$$

$$S_{BPSK}(t) = m(t)\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_c)$$

# Homework

- **9.4(a, b), 9.7, 9.10, 9.12, 9.13**